Week 7

Reading:

- *Geometry, Euclid and Beyond*: Chapter 2 Section 2

Exercises/questions:

1. Define the *projective plane* $\mathbb{P}^2$ to be the collection of lines in $\mathbb{R}^3$ through the origin. Define the lines of $\mathbb{P}^2$ to be planes in $\mathbb{R}^3$ containing the origin.

   (a) Show that $\mathbb{P}^2$ satisfies Hilbert's axioms of incidence.

   (b) Does $\mathbb{P}^2$ have parallel lines? Why or why not?

2. Let $(x, y, z), (x', y', z') \in \mathbb{R}^3 \setminus (0, 0, 0)$. Write $(x, y, z) \sim (x', y', z')$ if $(x', y', z') = \lambda(x, y, z)$ for some non-zero $\lambda \in \mathbb{R}$.

   (a) Show that $\sim$ is an equivalence relation.

   (b) Denote the equivalence class for (x, y, z) by $[x : y : z]$. Show that $\mathbb{P}^2$ is equivalent to the space

   - whose points are the equivalence classes $[x : y : z]$ and
   - whose lines are collections of points satisfying some linear equation $ax + by + cz = 0$.

   (c) Write $U_z = \{[x : y : z] \in \mathbb{P}^2 | z \neq 0\}$. Show that U_z is isomorphic to $\mathbb{R}^2$. Specifically, by isomorphism we mean a map $\varphi : U_z \to \mathbb{R}^2$ that gives a bijection between both the collection of points and the collection of lines.